Analysis of the equations of mathematical physics and foundations of field theories with the help of skew-symmetric differential forms

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Abstract

In the paper it is shown that, even without a knowledge of the concrete form of the equations of mathematical physics and field theories, with the help of skew-symmetric differential forms one can see specific features of the equations of mathematical physics, the relation between mathematical physics and field theory, to understand the mechanism of evolutionary processes that develop in material media and lead to emergency of physical structures forming physical fields. This discloses a physical meaning of such concepts like "conservation laws", "postulates" and "causality" and gives answers to many principal questions of mathematical physics and general field theory.

In present paper, beside the exterior forms, the skew-symmetric differential forms, whose basis (in contrast to the exterior forms) are deforming manifolds, are used. Mathematical apparatus of such differential forms (which were named evolutionary ones) includes nontraditional elements like nonidentical relations and degenerate transformations and this enables one to describe discrete transitions, quantum steps, evolutionary processes, and generation of various structures.

Introduction

Skew-symmetric differential forms possess unique properties that enable one to carry out a qualitative investigation of the equations of mathematical physics and the foundations of field theories. They can describe a conjugacy of various operators and objects (derivatives, differential equations, and so on).

Such a potentiality of skew-symmetric differential forms is due to the fact that skew-symmetric differential forms, as opposed to differential equations, deal with differentials and differential expressions rather than with derivatives.

In the paper, beside the exterior skew-symmetric differential forms that can describe conjugated objects, the skew-symmetric differential forms, whose basis (in contrast to the exterior forms) are deforming manifolds, are used. Such skew-symmetric differential forms, which were named evolutionary ones, can describe the process of conjugating objects and obtaining conjugated objects - the closed exterior forms. Evolutionary forms are obtained from the equations modelling physical processes, and therefore, they possess evolutionary properties.

The physical meaning of exterior skew-symmetric differential forms is connected with the fact that they correspond to conservation laws. Closed (inexact) exterior forms and relevant dual forms compose conjugated objects (a differential-geometrical structure). Just such conjugated objects, which are invariant ones, correspond to conservation laws. These are conservation laws for physical fields. The physical structures that form physical fields are just such conjugated objects.

The theory of closed exterior forms lies at the basis of field theories (the theories describing physical fields). The invariant properties of exterior forms explicitly or implicitly manifest themselves essentially in all formalisms of field theory, such as the Hamilton formalism, tensor approaches, group methods, quantum mechanics equations, the Yang-Mills theory and others. The gauge transformations (unitary, gradient and so on), the gauge symmetries and the identical relations of field theories are transformations, symmetries and relations of the theory of closed exterior forms.

In the paper it will be shown that the closed exterior forms, whose properties lie at the basis of field theories, are obtained from evolutionary forms related to the equations of mathematical physics. This discloses a relation between mathematical physics and the invariant field theories.

Evolutionary forms, as well as closed exterior forms, reflect the properties of conservation laws. However, they are conservation laws for material media. They are balance (differential) conservation laws. They are conservation laws for energy, linear momentum, angular momentum, and mass.

Below, on the basis of the properties of evolutionary forms it will be shown the noncommutativity of the balance conservation laws and their controlling role in the evolutionary processes developing in material media. In will be shown that such processes lead to origination of physical structures from which physical fields are formatted. During this the origination of physical structures in these processes reveals as an emergency of such formations like waves, vortices, turbulent pulsations, massless particles, and so on.

Such results firstly proves that material media generate physical fields. And secondly, they explain the nature of turbulence and various types of instabilities developed in material media.

Below it will be shown that the parameters of exterior and evolutionary forms allow to introduce a classification of physical fields and interactions.

The methods of investigating concrete material systems and physical fields on the basis of the exterior and evolutionary differential forms are demonstrated by the examples that are presented in Appendices 2.

The existence of evolutionary skew-symmetric differential forms has been established by the author while studying the problems of stability. Mathematical apparatus of evolutionary skew-symmetric differential forms includes nontraditional elements like nonidentical relations and degenerate transformations and this enables one to describe discrete transitions, quantum steps, evolutionary processes, and generation of various structures. Such mathematical apparatus is beyond the scope of existing physical and mathematical formalisms, and this makes difficulties in presentation and perception of this matter.

Closed exterior forms. Conservation laws

It is known that the exterior differential form of degree p (p-form) can be written as [1,2]

$$\theta^p = \sum_{i_1 \dots i_p} a_{i_1 \dots i_p} dx^{i_1} \wedge dx^{i_2} \wedge \dots \wedge dx^n \quad 0 \le p \le n \tag{1}$$

Here $a_{i_1 \dots i_p}$ are functions of the variables x^1, x^2, $\dots$, x^n, n is the dimension of space, $\wedge$ is the operator of exterior multiplication, dx^i, $dx^i \wedge dx^j$, $dx^i \wedge dx^j \wedge dx^k$, $\dots$ is the local basis which satisfies the condition of exterior multiplication (the condition of skew-symmetry).

The differential of the exterior form θ^p is expressed as

$$d\theta^p = \sum_{i_1 \dots i_p} da_{i_1 \dots i_p} \wedge dx^{i_1} \wedge dx^{i_2} \wedge \dots \wedge dx^{i_p} \tag{2}$$

The physical sense have the closed exterior forms. [Below we present only the data on exterior forms which are necessary for further presentation].

From the closure condition of the exterior form θ^p:

$$d\theta^p = 0 \tag{3}$$

one can see that the closed exterior form θ^p is a conserved quantity. This means that this can correspond to a conservation law, namely, to some conservative quantity.

If the form is closed only on pseudostructure, i.e. this form is a closed inexact one, the closure conditions are written as

$$d_\pi \theta^p = 0 \tag{4}$$

$$d_\pi {}^*\theta^p = 0 \tag{5}$$

where ${}^*\theta^p$ is the dual form.

Condition (5), i.e. the closure condition for dual form, specifies the pseudostructure π. {Cohomology (de Rham cohomology, singular cohomology and so on), sections of cotangent bundles, the surfaces eikonals, potential surfaces, pseudo-Riemannian and pseudo-Euclidean spaces, and others are examples of the psedustructures and manifolds that are formed by pseudostructures.}

From conditions (4) and (5) one can see the following. The dual form (pseudostructure) and closed inexact form (conservative quantity) made up a conjugated conservative object that can also correspond to some conservation law. The conservative object, which corresponds to the conservation law, is a differential-geometrical structure. (Such differential-geometrical structures are examples of G-structures.) The physical structures, which made up physical fields, and corresponding conservation laws are just such structures.

Properties of closed exterior differential forms.

1. *Invariance.*

It is known that the closed exact form is a differential of the form of lower degree:

$$\theta^p = d\theta^{p-1} \tag{6}$$

Closed inexact form is also a differential, and yet not a total one but an interior on pseudostructure

$$\theta^p_\pi = d_\pi \theta^{p-1} \tag{7}$$

Since the closed form is a differential (a total one if the form is exact, or an interior one on the pseudostructure if the form is inexact), it is obvious that the closed form turns out to be invariant under all transformations that conserve the differential. The unitary transformations (0-form), the tangent and canonical transformations (1-form), the gradient transformations (2-form) and so on are examples of such transformations. *These are gauge transformations for spinor, scalar, vector, tensor (3-form) fields.* It should be pointed out that just such transformations are used in field theory.

With the invariance of closed forms it is connected the covariance of relevant dual forms.

2. *Conjugacy. Duality. Symmetries.*

The closure of exterior differential forms is the result of conjugating the elements of exterior or dual forms. The closure property of the exterior form means that any objects, namely, the elements of exterior form, the components of elements, the elements of the form differential, the exterior and dual forms, the forms of sequential degrees and others, turn out to be conjugated.

With the conjugacy it is connected the duality.

The example of a duality having physical sense: the closed exterior form is a conservative quantity corresponding to conservation law, and the closed form (as the differential) can correspond to a certain potential force. (Below it will be shown in respect to what the closed form manifests itself as a potential force and with what the conservative physical quantity is connected).

The conjugacy is possible if there is one or another type of symmetry.

The gauge symmetries, which are interior symmetries of field theory and with which gauge transformations are connected, are symmetries of closed exterior differential forms. The conservation laws for physical fields are connected with such interior symmetries.

Identical relations of exterior forms.

Since the conjugacy is a certain connection between two operators or mathematical objects, it is evident that, to express the conjugacy mathematically, it can be used relations. These are identical relations.

The identical relations express the fact that each closed exterior form is the differential of some exterior form (with the degree less by one). In general form such an identical relation can be written as

$$d_\pi \varphi = \theta_\pi^p \tag{8}$$

In this relation the form in the right-hand side has to be a *closed* one.

Identical relations of exterior differential forms are a mathematical expression of various types of conjugacy that leads to closed exterior forms.

Such relations like the Poincare invariant, vector and tensor identical relations, the Cauchi-Riemann conditions, canonical relations, the integral relations by Stokes or Gauss-Ostrogradskii, the thermodynamic relations, the eikonal relations, and so on are examples of identical relations of closed exterior forms that have either the form of relation (8) or its differential or integral analogs.

One can see that identical relations of closed exterior differential forms make itself evident in various branches of physics and mathematics.

Below the mathematical and physical meaning of these relations will be disclosed with the help of evolutionary forms.

The analysis of field theories with the help of closed exterior forms.

The properties of closed exterior differential forms correspond to the conservation laws for physical fields. Therefore, the mathematical principles of the theory of closed exterior differential forms lie at the basis of existing field theories. {The physical fields [3] are a special form of the substance, they are carriers of various interactions such as electromagnetic, gravitational, wave, nuclear and other kinds ofinteractions. The conservation laws for physical fields are those that state an existence of conservative physical quantities or objects.}

The properties of closed exterior and dual forms, namely, invariance, covariance, conjugacy, and duality, lie at the basis of the group and structural properties of field theory.

The nondegenerate transformations of field theory are transformations of closed exterior forms. As it has been pointed out, the gauge transformations like the unitary, tangent, canonical, gradient and other gauge transformations are such transformations. These are transformations conserving the differential. Applications of nondegenerate transformations to identical relations enables one to find new closed exterior forms and, hence, to find new physical structures.

The gauge, i.e. internal, symmetries of the field theory equations are those of closed exterior forms.

The nondegenerate transformations of exterior differential forms lie at the basis of field theory operators. If, in addition to the exterior differential, we introduce the following operators: (1) δ for transformations that convert the form of $(p+1)$ degree into the form of p degree, (2) δ' for cotangent transformations, (3) Δ for the $d\delta - \delta d$ transformation, (4) Δ' for the $d\delta' - \delta' d$ transformation, one can write down the operators in the field theory equations in terms of these operators that act on the exterior differential forms. The operator δ corresponds to Green's operator, δ' does to the canonical transformation operator, Δ does to the d'Alembert operator in 4-dimensional space, and Δ' corresponds to the Laplace operator.

It can be shown that the equations of existing field theories are those obtained on the basis of the properties of the exterior form theory. The Hamilton formalism is based on the properties of closed exterior form of the first degree and corresponding dual form. The closed exterior differential form $ds = -Hdt + p_j dq_j$ (the Poincare invariant) corresponds to the field equation related to the Hamilton system. The Schrődinger equation in quantum mechanics is an analog to field equation, where the conjugated coordinates are changed by operators. It is evident that the closed exterior form of zero degree (and dual form) correspond to quantum mechanics. Dirac's *bra-* and *cket-* vectors constitute a closed exterior form of zero degree [4]. The properties of closed exterior form of the second degree (and dual form) lie at the basis of the electromagnetic

field equations. The Maxwell equations may be written as [5] $d\theta^2 = 0$, $d^*\theta^2 = 0$, where $\theta^2 = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu$ (here $F_{\mu\nu}$ is the strength tensor). Closed exterior and dual forms of the third degree correspond to the gravitational field. (However, to the physical field of given type it can be assigned closed forms of less degree. In particular, to the Einstein equation for gravitational field it is assigned the first degree closed form, although it was pointed out that the type of a field with the third degree closed form corresponds to the gravitational field.)

The connection between the field theory equations and gauge transformations used in field theories with closed exterior forms of appropriate degrees shows that there exists a commonness between field theories describing physical fields of different types. This can serve as an approach to constructing the unified field theory. This connection shows that it is possible to introduce a classification of physical fields according to the degree of closed exterior form. Such a classification also exists for interactions (see below).

(But within the framework of only exterior differential forms one cannot understand how this classification is explained. This can be elucidated only by application of evolutionary differential forms.)

And here it should underline that the field theories are based on the properties of closed *inexact* forms. This is explained by the fact that only inexact exterior forms can correspond to the physical structures that form physical fields. The condition that the closed exterior forms, which constitute the basis of field theory equations, are inexact ones reveals in the fact that essentially all existing field theories include a certain elements of noninvariance. Such elements of noninvariance are, for example, nonzero value of the curvature tensor in Einstein's theory [6], the indeterminacy principle in Heisenberg's theory, the torsion in the theory by Weyl [6], the Lorentz force in electromagnetic theory [7], an absence of general integrability of the Schrődinger equations, the Lagrange function in the variational methods, an absence of the identical integrability of the mathematical physics equations and that of identical covariance of the tensor equations, and so on. Only if we assume the elements of noncovariance, we can obtain closed *inexact* forms that correspond to physical structures.

And yet, the existing field theories are invariant ones because they are provided with additional conditions under which the invariance or covariance requirements have to be satisfied. It is possible to show that these conditions are the closure conditions of exterior or dual forms. Examples of such conditions may be the identity relations: canonical relations in the Schrődinger equations, gauge invariance in electromagnetic theory, commutator relations in the Heisenberg theory, symmetric connectednesses, identity relations by Bianchi in the Einstein theory, cotangent bundles in the Yang-Mills theory, the Hamilton function in the variational methods, the covariance conditions in the tensor methods, etc.

It is known that the equations of existing field theories and the mathematical formalisms of field theories have been obtained on the basis of postulates. One can see that these postulates are obtained from the closure conditions of inexact exterior forms.

Thus one can see that the properties and mathematical apparatus of closed exterior forms made up the basis of existing field theories.

And here it arises the question of how closed inexact exterior forms, which correspond to physical structures and reflect the properties of conservation laws and on whose properties field theories are based, are obtained. This gives the answers to the following questions: (a) how the physical structures, from which physical fields are formatted, originate; (b) what generates physical structures; (c) how the process of generation proceeds, and (d) what is responsible for such processes? That is, this enables one to understand the heart of physical evolutionary processes and their causality. This has to explain both the internal connection between different physical fields and their classification.

Below it will be shown that the closed inexact exterior forms can be obtained from the evolutionary forms.

Distinction between exterior and evolutionary forms

Skew-symmetric differential forms, that the author named evolutionary ones, differ in their properties from exterior forms. The distinction between exterior and evolutionary skew-symmetric differential forms is connected with the properties of manifolds on which skew-symmetric forms are defined.

It is known that the exterior differential forms are skew-symmetric differential forms whose basis are differentiable manifolds or they can be manifolds with structures of any type [2,8]. (Such manifolds have one common property, namely, they locally admit one-to-one mapping into the Euclidean subspaces and into other manifolds or submanifolds of the same dimension [8]).

While describing the evolutionary processes in material systems (material media) one is forced to deal with manifolds which do not allow one-to-one mapping described above. Lagrangian manifolds and tangent manifolds of differential equations describing physical processes can be examples of deforming manifolds.

Such manifolds are those constructed of trajectories of the material system elements (particles). These manifolds, which can be called accompanying manifolds, are deforming variable manifolds.

The skew-symmetric differential forms defined on these manifolds are evolutionary ones. The coefficients of these differential forms and the characteristics of corresponding manifolds are interconnected and are varied as functions of evolutionary variables.

A distinction between manifolds on which exterior and evolutionary forms are defined relates to the properties of metric forms of these manifolds.

Below we present some information about the manifolds on which skew-symmetrical differential forms are defined.

Some properties of manifolds.

Assume that on the manifold one can set the coordinate system with base vectors $\mathbf{e}_\mu$ and define the metric forms of manifold [6]: $(\mathbf{e}_\mu\mathbf{e}_\nu)$, $(\mathbf{e}_\mu dx^\mu)$, $(d\mathbf{e}_\mu)$. The metric forms and their commutators define the metric and differential characteristics of the manifold.

If metric forms are closed (the commutators are equal to zero), the metric is defined $g_{\mu\nu} = (\mathbf{e}_\mu \mathbf{e}_\nu)$, and the results of translation over manifold of the point $d\mathbf{M} = (\mathbf{e}_\mu dx^\mu)$ and of the unit frame $d\mathbf{A} = (d\mathbf{e}_\mu)$ prove to be independent of the curve shape (the path of integration).

To describe the manifold differential characteristics and, correspondingly, the metric form commutators, one can use connectednesses [2,6]. If the components of metric form can be expressed in terms of connectedness $\Gamma^\rho_{\mu\nu}$ [6], the expressions $\Gamma^\rho_{\mu\nu}$, $(\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu})$ and $R^\mu_{\nu\rho\sigma}$ are components of the commutators of the metric forms with zeroth-, first-, and third degrees. (The commutator of the second degree metric form is written down in a more complex manner [6], and therefore it is not presented here).

The closed metric forms define the manifold structure. And the commutators of metric forms define the manifold differential characteristics that specify the manifold deformation: bending, torsion, rotation, and twist. (For example, the commutator of the zeroth degree metric form $\Gamma^\rho_{\mu\nu}$ characterizes the bend, that of the first degree form $(\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu})$ characterizes the torsion, the commutator of the third-degree metric form $R^\mu_{\nu\rho\sigma}$ determines the curvature. (For manifolds with closed metric form of first degree the coefficients of connectedness are symmetric ones.)

Is is evident that the manifolds which are metric ones or possess the structure have closed metric forms. It is with such manifolds the exterior differential forms are connected.

If the manifolds are deforming manifolds, this means that their metric form commutators are nonzero. That is, the metric forms of such manifolds turn out to be unclosed.

The evolutionary skew-symmetric differential forms are defined on manifolds with unclosed metric forms.

Specific properties of evolutionary skew-symmetric differential forms and the distinction of evolutionary forms from exterior ones are connected with the properties of commutators of unclosed metric form, which enter into the evolutionary form commutators.

Distinction between differentials of exterior and evolutionary forms.

The evolutionary differential form of degree p (p-form) is written similarly to exterior differential form. But the evolutionary form differential cannot be written similarly to that presented for exterior differential forms. In the evolutionary form differential there appears an additional term connected with the fact that the basis of evolutionary form changes. For differential forms defined on the manifold with unclosed metric form one has $d(dx^{\alpha_1} \wedge dx^{\alpha_2} \wedge \ldots \wedge dx^{\alpha_p}) \neq 0$. (For differential forms defined on the manifold with closed metric form one has $d(dx^{\alpha_1} \wedge dx^{\alpha_2} \wedge \ldots \wedge dx^{\alpha_p}) = 0$). For this reason the differential of the evolutionary form θ^p can be written as

$$d\theta^p = \sum_{\alpha_1 \ldots \alpha_p} da_{\alpha_1 \ldots \alpha_p} \wedge dx^{\alpha_1} \wedge dx^{\alpha_2} \ldots \wedge dx^{\alpha_p} + \sum_{\alpha_1 \ldots \alpha_p} a_{\alpha_1 \ldots \alpha_p} d(dx^{\alpha_1} \wedge dx^{\alpha_2} \ldots \wedge dx^{\alpha_p}) \tag{9}$$

where the second term is a differential of unclosed metric form of nonzero value.

[In further presentation the symbol of summing $\sum$ and the symbol of exterior multiplication $\wedge$ will be omitted. Summation over repeated indices will be implied.]

The second term connected with the differential of the basis can be expressed in terms of the metric form commutator.

For example, let us consider the first-degree form $\theta = a_\alpha dx^\alpha$. The differential of this form can be written as

$$d\theta = K_{\alpha\beta} dx^\alpha dx^\beta \tag{10}$$

where $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$ are the components of commutator of the form θ, and $a_{\beta;\alpha}$, $a_{\alpha;\beta}$ are covariant derivatives. If we express the covariant derivatives in terms of the connectedness (if it is possible), they can be written as $a_{\beta;\alpha} = \partial a_\beta/\partial x^\alpha + \Gamma^\sigma_{\beta\alpha} a_\sigma$, where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. We arrive at the following expression for the commutator components of the form θ

$$K_{\alpha\beta} = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta}\right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta}) a_\sigma \tag{11}$$

Here the expressions $(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})$ entered into the second term are just the components of commutator of the first-degree metric form.

If to substitute the expressions (11) for evolutionary form commutator into formula (10), we obtain the following expression for differential of the first degree skew-symmetric form

$$d\theta = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta}\right) dx^\alpha dx^\beta + \left((\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta}) a_\sigma\right) dx^\alpha dx^\beta \tag{12}$$

The second term in the expression for differential of skew-symmetric form is connected with the differential of the manifold metric form, which is expressed in terms of the metric form commutator.

Thus, the differentials and, correspondingly, the commutators of exterior and evolutionary forms are of different types. In contrast to the exterior form commutator, the evolutionary form commutator includes two terms. These two terms have different nature, namely, one term is connected with the coefficients of evolutionary form itself, and the other term is connected with differential characteristics of manifold. Interaction between terms of the evolutionary form commutator (interactions between coefficients of evolutionary form and its basis) provides the foundation of evolutionary processes that lead to generation of closed inexact exterior forms.

Properties of evolutionary forms.

Above it has been shown that the evolutionary form commutator includes the commutator of the manifold metric form which is nonzero. Therefore, the evolutionary form commutator cannot be equal to zero. This means that the

evolutionary form differential is nonzero. Hence, the evolutionary form, in contrast to the case of the exterior form, cannot be closed. This leads to that in the mathematical apparatus of evolutionary forms there arise new unconventional elements like nonidentical relations and degenerate transformations. Just such peculiarities allow to describe evolutionary processes.

Nonidentical relations of the evolutionary form theory, as well as identical relations of the theory of closed exterior forms, are relations between the differential and the skew-symmetric form. In the right-hand side of the identical relation of exterior forms (see relation (8)) it stands a closed form, which is a differential as well as the left-hand side. And in the right-hand of the nonidentical relation of evolutionary form it stands the evolutionary form that is not closed and, hence, cannot be a differential like the left-hand side. Such a relation cannot be identical one.

Nonidentical relations are obtained while describing any processes in terms of differential equations. (In Appendix 1 we present an example of a qualitative investigation of differential equations.) The relation of such type is obtained, for example, while analyzing the integrability of the partial differential equation. The equation is integrable if it can be reduced to the form $d\phi = dU$. However it appears that, if the equation is not subject to an additional condition (the integrability condition), the right-hand side turns out to be an unclosed form and it cannot be expressed as a differential.

Nonidentical relations of evolutionary forms are evolutionary relations because they include the evolutionary form. Such nonidentical evolutionary relations appear to be selfvarying ones. A variation of any object of the relation in some process leads to a variation of another object and, in turn, a variation of the latter leads to a variation of the former. Since one of the objects is a noninvariant (i.e. unmeasurable) quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot be completed.

The nonidentity of evolutionary relation is connected with a nonclosure of the evolutionary form, that is, it is connected with the fact that the evolutionary form commutator is nonzero. As it has been pointed out, the evolutionary form commutator includes two terms: one term specifies the mutual variations of the evolutionary form coefficients, and the second term (the metric form commutator) specifies the manifold deformation. These terms have a different nature and cannot make the commutator vanish. In the process of selfvariation of the nonidentical evolutionary relation it proceeds an exchange between the terms of the evolutionary form commutator and this is realized according to the evolutionary relation. The evolutionary form commutator describes a quantity that is a moving force of the evolutionary process.

The significance of the evolutionary relation selfvariation consists in the fact that in such a process it can be realized the conditions of degenerate transformation under which the closed inexact exterior form is obtained from the evolutionary form, and from nonidentical relation the identical relation is obtained.

Analysis of the mathematical physics equations describ-

ing physical processes in material media with the help of evolutionary forms

Exterior and evolutionary forms enable one to investigate the integrability of differential equations. This is due to the fact that they make it possible to study the conjugacy of the equations or their derivatives. The type of solutions to differential equations depends on the conjugacy. Solutions are invariant if the equations and their derivatives are conjugated ones. If this is not fulfilled, the solutions prove to be noninvariant, namely, they are functionals rather then functions.

The qualitative analysis of the equations of mathematical physics with the help of differential forms shows that any differential equations describing any processes turn out to be nonintegrable without additional conditions. Additional conditions under which the equations become integrable can be realized only discretely. This points to the fact that the solutions of any differential equations of mathematical physics describing physical processes can be only generalized (discrete) ones (see Appendix 1). These are precisely generalized solutions that describe various structures.

The importance of evolutionary forms consists in the fact that they allow not only to investigate an integrability of the equations and functional properties of the solutions. They also allow to describe the process of realization of invariant solutions in itself and thereby to describe the process of *origination* of physical structures and to disclose the mechanism of processes like turbulence, generation of waves and vortices, creation of massless particles, and so on.

Below we will carry out the analysis of the equations of mathematical physics, which describe physical processes in material media. These are precisely material media that generate physical structures making up physical fields.

It will be shown that the closed exterior forms, which correspond to the conservation laws *for physical fields* and describe physical structures, are obtained from the evolutionary forms that are connected with the equations of conservation laws *for material media*. The conservation laws for material media are balance (differential) conservation laws. The process of obtaining closed exterior forms from evolutionary ones just describes the process of generating physical structures by material media. These conclusions follow from the analysis of the equations of balance conservation laws with the help of differential forms.

[Sometimes below it will be used a double notation in subtitles, one in reference to physical meaning and another in reference to mathematical meaning.]

Connection of evolutionary forms with the balance conservation laws.

The balance conservation laws are those that establish the balance between the variation of a physical quantity and the corresponding external action. These are the conservation laws for material systems (material media) [9].

The balance conservation laws are the conservation laws for energy, linear momentum, angular momentum, and mass.

The equations of the balance conservation laws are differential (or integral) equations that describe a variation of functions corresponding to physical quan-

tities [10-13]. (The specific forms of these equations for thermodynamical and gas dynamical material systems and the systems of charged particles will be presented in the Appendix 2).

But it appears that, even without a knowledge of the concrete form of these equations, with the help of the differential forms one can see specific features of these equations that elucidate the properties of the balance conservation laws. To do so it is necessary to study the conjugacy (consistency) of these equations.

Equations are conjugate if they can be contracted into identical relations for the differential, i.e. for a closed form.

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this system is connected with the manifold built by the trajectories of the material system elements). The energy equation in the inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A$$

where D/Dt is the total derivative with respect to time, ψ is the functional of the state that specifies the material system (every material system has its own functional of the state), A is the quantity that depends on specific features of the system and on external energy actions onto the system. {The action functional, entropy, wave function can be regarded as examples of the functional ψ. Thus, the equation for energy presented in terms of the action functional S has a similar form: $DS/Dt = L$, where $\psi = S$, $A = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of an ideal gas can be presented in the form [13]: $Ds/Dt = 0$, where s is entropy. In this case $\psi = s$, $A = 0$. It is worth noting that the examples presented show that the action functional and entropy play the same role.}

In the accompanying frame of reference the total derivative with respect to time is transformed into the derivative along the trajectory. Equation of energy is now written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1 \tag{13}$$

Here ψ is the functional specifying the state of material system, ξ^1 is the coordinate along the trajectory, A_1 is the quantity that depends on specific features of material system and on external (with respect to local domain of material system) energy actions onto the system.

In a similar manner, in the accompanying reference system the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu, \quad \nu = 2, ... \tag{14}$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of material system and on external force actions.

Eqs. (13) and (14) can be convoluted into the relation

$$d\psi \,=\, A_\mu\, d\xi^\mu, \quad (\mu \,=\, 1,\, \nu) \tag{15}$$

where $d\psi$ is the differential expression $d\psi \,=\, (\partial\psi/\partial\xi^\mu)d\xi^\mu$.

Relation (15) can be written as

$$d\psi \,=\, \omega \tag{16}$$

here $\omega \,=\, A_\mu\, d\xi^\mu$ is the skew-symmetrical differential form of the first degree.

Since the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (16) has been obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be a form of the second degree. And in combination with the equation of the balance conservation law for mass this form will be a form of degree 3.

Thus, in general case the evolutionary relation can be written as

$$d\psi \,=\, \omega^p \tag{17}$$

where the form degree p takes the values $p \,=\, 0, 1, 2, 3..$ (The evolutionary relation for $p \,=\, 0$ is similar to that in the differential forms, and it was obtained from the interaction of energy and time.)

Let us show that relation obtained from the equation of the balance conservation laws proves to be nonidentical.

To do so we shall analyze relation (16).

In the left-hand side of relation (16) there is a differential that is a closed form. This form is an invariant object. The right-hand side of relation (16) involves the differential form ω that is not an invariant object because in real processes, as it will be shown below, this form proves to be unclosed. The commutator of this form is nonzero. The components of commutator of the form $\omega \,=\, A_\mu d\xi^\mu$ can be written as follows:

$$K_{\alpha\beta} \,=\, \left(\frac{\partial A_\beta}{\partial \xi^\alpha} \,-\, \frac{\partial A_\alpha}{\partial \xi^\beta}\right)$$

(here the term connected with the manifold metric form has not yet been taken into account).

The coefficients A_μ of the form ω have to be obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω consisted of the derivatives of such

coefficients is nonzero. This means that the differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed and cannot be a differential like the left-hand side. This means that relation (16) cannot be an identical one.

In a similar manner one can prove the nonidentity of relation (17).

Hence, without a knowledge of particular expression for the form ω, one can argue that for actual processes the relation obtained from the equations corresponding to the balance conservation laws proves to be nonidentical.

Physical meaning of nonidentical evolutionary relation.

The nonidentity of evolutionary relation means that the balance conservation law equations are inconsistent. And this indicates that the balance conservation laws are noncommutative. (If the balance conservation laws be commutative, the equations would be consistent and the evolutionary relation would be identical).

To what such a noncommutativity of the balance conservation laws leads?

Nonidentical evolutionary relation obtained from the equations of the balance conservation laws involves the functional that specifies the material system state. However, since this relation turns out to be not identical, from this relation one cannot get the differential $d\psi$ that could point out to the equilibrium state of material system. The absence of differential means that the system state is nonequilibrium. That is, due to noncommutativity of the balance conservation laws the material system state turns out to be nonequilibrium under effect of external actions. This points out to the fact that in material system the internal force acts. (External actions onto local domain of material system lead to emergency of internal forces in local domain.)

The action of internal force leads to a distortion of trajectories of material system. The manifold made up by the trajectories (the accompanying manifold) turns out to be a deforming manifold. The differential form ω, as well as the forms ω^p defined on such manifold, appear to be evolutionary forms. Commutators of these forms will contain an additional term connected with the commutator of unclosed metric form of manifold, which specifies the manifold deformation.

Selfvariation of nonidentical evolutionary relation. (Selfvariation of nonequilibrium state of material system.)

The availability of two terms in the commutator of the form ω^p and the nonidentity of evolutionary relation lead to that the relation obtained from the balance conservation law equations turns out to be a selfvarying relation.

Selfvariation of nonidentical evolutionary relation points to the fact that the nonequilibrium state of material system turns out to be selfvarying. State of material system changes but remains nonequilibrium during this process.

It is evident that this selfvariation proceeds under the action of internal force whose quantity is described by the commutator of the unclosed evolutionary form ω^p. (If the commutator be zero, the evolutionary relation would be identical, and this would point to the equilibrium state, i.e. the absence of in-

ternal forces.) Everything that gives a contribution into the commutator of the form ω^p leads to emergency of internal force.

What is the result of such a process of selfvarying the nonequilibrium state of material system?

Degenerate transformation. Emergency of closed exterior forms. (Origination of physical structures.)

The significance of the evolutionary relation selfvariation consists in the fact that in such a process it can be realized conditions under which the closed exterior form is obtained from the evolutionary form.

These are conditions of degenerate transformation. Since the differential of evolutionary form, which is unclosed, is nonzero, but the differential of closed exterior form equals zero, the transition from evolutionary form to closed exterior form is possible only as a degenerate transformation, namely, a transformation that does not conserve the differential. And this transition is possible exclusively to closed *inexact* exterior form, i.e. to the external form being closed on pseudostructure. The conditions of degenerate transformation are those of vanishing the commutator (interior one) of the metric form defining the pseudostructure, in other words, the closure conditions for dual form.

As it has been already mentioned, the evolutionary differential form ω^p, involved into nonidentical relation (17) is an unclosed one. The commutator of this form, and hence the differential, are nonzero. That is,

$$d\omega^p \neq 0 \tag{18}$$

If the conditions of degenerate transformation are realized, then from the unclosed evolutionary form one can obtain the differential form closed on pseudostructure. The differential of this form equals zero. That is, it is realized the transition $d\omega^p \neq 0 \rightarrow$ (degenerate transformation) $\rightarrow d_\pi \omega^p = 0$, $d_\pi{}^*\omega^p = 0$

The relations obtained

$$d_\pi \omega^p = 0, d_\pi{}^*\omega^p = 0 \tag{19}$$

are the closure conditions for exterior inexact form. This means that it is realized the exterior form closed on pseudostructure.

The realization of closed (on pseudostructure) inexact exterior form points to emergency of physical structure [14].

To the degenerate transformation it must correspond a vanishing of some functional expressions, such as Jacobians, determinants, the Poisson brackets, residues and others. Vanishing these functional expressions is the closure condition for dual form.

The conditions of degenerate transformation that lead to emergency of closed inexact exterior form are connected with any symmetries. Since these conditions are conditions of vanishing the interior differential of the metric form, i.e. vanishing the interior (rather than total) metric form commutator, the conditions of degenerate transformation can be caused by symmetries of coefficients

of the metric form commutator (for example, these can be symmetrical connectednesses).

While describing material system the symmetries can be due to degrees of freedom of material system. The translational degrees of freedom, internal degrees of freedom of the system elements, and so on can be examples of such degrees of freedom.

And it should be emphasized once more that *the degenerate transformation is realized as a transition from the accompanying noninertial coordinate system to the locally inertial system*. The evolutionary form is defined in the noninertial frame of reference (deforming manifold). But the closed exterior form formatted is obtained with respect to the locally-inertial frame of reference (pseudostructure).

The conditions of degenerate transformation (vanishing the dual form commutator) define a pseudostructure. These conditions specify the derivative of implicit function, which defines the direction of pseudostructure.

The speeds of various waves are examples of such derivatives: the speed of light, the speed of sound and of electromagnetic waves (see the Appendix), the speed of creating particles and so on.

It can be shown that the equations for surfaces of potential (of simple layer, double layer), equations for one, two, ... eikonals, of the characteristic and of the characteristic surfaces, the residue equations and so on serve as the equations for pseudostructures.

The mechanism of creating the pseudostructures lies at the basis of forming the pseudometric surfaces and their transition into the metric spaces (see below).

Obtaining identical relation from nonidentical one. (Transition of material system into a locally equilibrium state.)

On the pseudostructure π evolutionary relation (17) converts into the relation

$$d_\pi \psi = \omega_\pi^p \tag{20}$$

which proves to be an identical relation. Indeed, since the form ω_π^p is a closed one, on the pseudostructure this form turns out to be a differential of some differential form. In other words, this form can be written as $\omega_\pi^p = d_\pi \theta$. Relation (20) is now written as

$$d_\pi \psi = d_\pi \theta$$

There are differentials in the left-hand and right-hand sides of this relation. This means that the relation is an identical one.

Thus one can see that under degenerate transformation from evolutionary relation the relation identical on pseudostructure is obtained. Here it should be emphasized that in this case the evolutionary relation itself remains to be nonidentical one. The differential, which equals zero, is an interior one. The evolutionary form commutator vanishes only on pseudostructure. The total evolutionary form commutator is nonzero. That is, under degenerate transformation the evolutionary form differential vanishes only on pseudostructure.

The total differential of the evolutionary form is nonzero. The evolutionary form remains to be unclosed.

From relation (20) one can obtain a differential which specifies the state of material system (and the state function), and this corresponds to equilibrium state of the system. But identical relation can be realized only on pseudostructure (which is specified by the condition of degenerate transformation). This means that the transition of material system to equilibrium state proceeds only locally (in the local domain of material system). In other words, it is realized the transition of material system from nonequilibrium state to locally equilibrium one. In this case the global state of material system remains to be nonequilibrium.

The transition from nonidentical relation (17) obtained from the balance conservation laws to identical relation (20) means the following. Firstly, an emergency of the closed (on pseudostructure) inexact exterior form (right-hand side of relation (20)) points to an origination of physical structure. And, secondly, an existence of the state differential (left-hand side of relation (20)) points to a transition of material system from nonequilibrium state to the locally-equilibrium state.

Thus one can see that the transition of material system from nonequilibrium state to locally-equilibrium state is accompanied by originating differential-geometrical structures, which are physical structures.

The emergency of physical structures in the evolutionary process reveals in material system as an emergency of certain observable formations, which develop spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, creating massless particles, and others. The intensity of such formations is controlled by a quantity accumulated by the evolutionary form commutator at the instant in time of originating physical structures.

Here the following should be pointed out. Physical structures are generated by local domains of material system. They are elementary physical structures. By combining with one another they can form large-scale structures and physical fields.

The availability of physical structures points out to fulfilment of conservation laws. These are conservation laws for physical fields. The process of generating physical structures (forming physical fields) demonstrates a connection of these conservation laws, which had been named as exact ones, with the balance (differential) conservation laws for material media. The physical structures that correspond to the exact conservation laws are produced by material system in the evolutionary processes, which are based on the interaction of noncommutative balance conservation laws.

Noncommutativity of balance conservation laws for material media and their controlling role in evolutionary processes accompanied by emerging physical structures practically have not been taken into account in the explicit form anywhere. The mathematical apparatus of evolutionary differential forms enables one to take into account and to describe these points [9].

Physical meaning of the duality of closed exterior forms as conservative quantities and as potential forces. (Potential forces)

The duality of closed exterior forms as conservative quantities and as potential forces points to that an unmeasurable quantity, which is described by the evolutionary form commutator (recall, that all external, with respect to local domain, actions make a contribution into this commutator) and acts as an internal force, is converted into a measurable quantity that acts as a potential force.

Where, from what, and on what the potential force acts?

The potential force is an action of created (quantum) formation onto the local domains of the material system over which it is translated. And if the internal force acts in the interior of the local domain of material system (and it caused that to deform), the potential force acts onto the neighboring domain. The local domain gets rid of its internal force and modifies that into a potential force which acts onto neighboring domains. An unmeasurable quantity, that acts in local domain as an internal force, is transformed into a measurable quantity of the observable formation (and the physical structure as well) that is emitted from the local domain and acts onto neighboring domain as a force equal to this quantity.

If the external actions equal zero (the evolutionary form commutator be equal to zero), then internal and potential forces equal zero.

Thus, one has to distinguish the forces of three types: 1) external forces (the actions being external with respect to local domain), 2) internal forces that originate in local domains of material system due to the fact that the physical quantities of material system changed by external actions turn out to be inconsistent, and 3) the potential forces are forces of the action of the formations (corresponding to physical structures) onto material system.

The potential force, whose value is conditioned by the quantity of the commutator of the evolutionary form ω^p at the instant of the formation production, acts normally to the pseudostructure, i.e. with respect to the integrating direction, along which the interior differential (the closed form) is formed. The potential forces are described, for example, by jumps of derivatives in the direction normal to characteristics, to potential surfaces and so on. This corresponds to the fact that the evolutionary form commutators along these directions are nonzero.

The duality of closed inexact form as a conservative quantity and as a potential force shows that potential forces are the action of formations corresponding to physical structures onto material system.

Connection of the characteristics of the structures originated with the characteristics of differential forms. (Characteristics of physical structures and the formation created)

Since the closed inexact exterior form corresponding to physical structure was obtained from the evolutionary form, it is evident that the characteristics of physical structure originated has to be connected with those of the evolutionary form and of the manifold on which this form is defined as well as with the

conditions of degenerate transformation and with the values of commutators of the evolutionary form and the manifold metric form.

The conditions of degenerate transformation, i.e. symmetries caused by degrees of freedom of material system, determine the equation for pseudostructures.

The closed exterior forms corresponding to physical structures are conservative quantities. These conservative quantities describe certain charges.

The first term of the commutator of evolutionary form determines the value of discrete change (the quantum), which the quantity conserved on the pseudostructure undergoes during transition from one pseudostructure to another. The second term of the evolutionary form commutator specifies a characteristics that fixes the character of the manifold deformation, which took place before physical structure emerged. (Spin is an example of such a characteristics).

Characteristics of physical structures depends in addition on the properties of material system generating these structures.

The closed exterior forms obtained correspond to the state differential for material system. The differentials of entropy, action, potential and others are examples of such differentials.

As it was already mentioned, in material system the created physical structure is revealed as an observable formation. It is evident that the characteristics of the formation, as well as those of created physical structure, are determined by the evolutionary form and its commutator and by the material system characteristics.

It can be shown that the following correspondence between characteristics of the formations emerged and characteristics of evolutionary forms, of the evolutionary form commutators and of material system is established:

1) an intensity of the formation (a potential force) $\leftrightarrow$ *the value of the first term in the commutator of evolutionary form* at the instant when the formation is created;

2) vorticity (an analog of spin) $\leftrightarrow$ *the second term in the commutator that is connected with the metric form commutator*;

3) an absolute speed of propagation of created formation (the speed in the inertial frame of reference) $\leftrightarrow$ *additional conditions connected with degrees of freedom of material system*;

4) a speed of the formation propagation relative to material system $\leftrightarrow$ *additional conditions connected with degrees of freedom of material system plus the velocity of elements of local domain.*

Parameters of differential forms. (Classification of physical structures)

The connection of physics structures with skew-symmetric differential forms allows to introduce a classification of these structures in dependence on parameters that specify skew-symmetric differential forms and enter into nonidentical and identical relation. To determine these parameters one has to consider the problem of integration of nonidentical evolutionary relation.

Since the identical relation obtained from nonidentical evolutionary relation contains only differential and the closed form also is a differential, one can integrate (on pseudostructure) this relation and obtain a relation with the differential forms of less by one degree. From the relation obtained, which will be nonidentical one, under degenerate transformation it can be obtained new identical relation that can be integrated once more.

Thus, from the nonidentical relation, which contains the evolutionary form of degrees p, it can be obtained identical relations with closed inexact forms of degrees k, where k ranges from p to 0. That is, evolutionary forms of degree p can generate closed inexact forms of degrees $k = p$, $k = p-1$, $\dots$, $k = 0$. Under degenerate conditions from closed inexact forms of zero degree it is obtained an exact exterior form of zero degree which the metric structure corresponds to.

In addition to these parameters, another parameter appears, namely, the dimension of space n If the evolutionary relation generates the closed forms of degrees $k = p$, $k = p-1$, $\dots$, $k = 0$, to them there correspond the pseudostructures of dimensions $(n + 1 - k)$.

The parameters of evolutionary and exterior forms that follow from the evolutionary forms allow to introduce a classification of physical structures that defines a type of physical structures and, accordingly, of physical fields and interactions.

The type of physical structures (and, accordingly, of physical fields) generated by the evolutionary relation depends on the degree of differential forms p and k and on the dimension of original inertial space n. (Here p is the degree of evolutionary form in nonidentical relation that is connected with a number of interacting balance conservation laws, and k is the degree of closed form generated by nonidentical relation. Recall that the interaction of balance conservation laws for energy and linear momentum corresponds to the value $p = 1$, with the balance conservation law for angular momentum in addition this corresponds to the value $p = 2$, and with the balance conservation law for mass in addition it corresponds to the value $p = 3$. The value $p = 0$ corresponds to interaction between time and the balance conservation law for energy.)

By introducing a classification by numbers p, k, n one can understand the internal connection between various physical fields. Since physical fields are the carriers of interactions, such classification enables one to see a connection between interactions.

On the basis of the properties of evolutionary forms that correspond to the conservation laws, one can suppose that such a classification may be presented in the form of the table given below. This table corresponds to elementary particles.

{It should be emphasized the following. Here the concept of "interaction" is used in a twofold meaning: an interaction of the balance conservation laws that relates to material systems, and the physical concept of "interaction" that relates to physical fields and reflects the interactions of physical structures, namely, it is connected with exact conservation laws}.

TABLE

interaction	$k\backslash p,n$	**0**	**1**	**2**	**3**
gravitation	**3**				**graviton** $\Uparrow$ electron proton neutron photon
electro-magnetic	**2**			**photon2** $\Uparrow$ electron proton neutrino	**photon3**
weak	**1**		**neutrino1** $\Uparrow$ electron quanta	**neutrino2**	**neutrino3**
strong	**0**	**quanta0** $\Uparrow$ quarks?	**quanta1**	**quanta2**	**quanta3**
particles material nucleons?	exact forms	**electron**	**proton**	**neutron**	**deuteron?**
N		1 time	2 time+ 1 coord.	3 time+ 2 coord.	4 time+ 3 coord.

In the Table the names of the particles created are given. Numbers placed near particle names correspond to the space dimension. Under the names of particles the sources of interactions are presented. In the next to the last row we present particles with mass (the elements of material system) formed by interactions (the exact forms of zero degree obtained by sequential integrating the evolutionary relations with the evolutionary forms of degree p corresponding to these particles). In the bottom row the dimension of the *metric* structure created is presented.

From the Table one can see the correspondence between the degree k of the closed forms realized and the type of interactions. Thus, $k = 0$ corresponds to strong interaction, $k = 1$ corresponds to weak interaction, $k = 2$ corresponds to electromagnetic interaction, and $k = 3$ corresponds to gravitational interaction. The degree k of the closed forms realized and the number of interacting balance conservation laws determine the type of interactions and the type of particles created. The properties of particles are governed by the space dimension. The last property is connected with the fact that closed forms of equal degrees k, but obtained from the evolutionary relations acting in spaces of different dimensions n, are distinctive because they are defined on pseudostructures of different dimensions (the dimension of pseudostructure $(n+1-k)$ depends on the dimension of initial space n). For this reason the realized physical structures with

closed forms of equal degrees k are distinctive in their properties.

The parameters p, k, n can range from 0 to 3. They determine some completed cycle. The cycle involves four levels, to each of which there correspond their own values of p ($p = 0, 1, 2, 3$) and space dimension n.

In the Table one cycle of forming physical structures is presented. Each material system has his own completed cycle. This distinguishes one material system from another system. One completed cycle can serve as the beginning of another cycle (the structures formed in the preceding cycle serve as the sources of interactions for beginning a new cycle). This may mean that one material system (medium) proves to be imbedded into the other material system (medium). The sequential cycles reflect the properties of sequentially imbedded material systems. And yet a given level has specific properties that are inherent characteristics of the same level in another cycles. This can be seen, for example, from comparison of the cycle described and the cycle in which to the exact forms there correspond conductors, semiconductors, dielectrics, and neutral elements. The properties of elements of the third level, namely, of neutrons in one cycle and of dielectrics in another, are identical to the properties of so called "magnetic monopole" [15,16].

Forming pseudometric and metric spaces

The mechanism of creating the pseudostructures lies at the basis of forming the pseudometric surfaces and their transition into metric spaces [17]. (It should be pointed out that the eigenvalues and the coupling constants appear as the conjugacy conditions for exterior or dual forms, the numerical constants are the conjugacy conditions for exact forms.)

It was shown above that the evolutionary relation of degree p can generate (in the presence of degenerate transformations) closed forms of the degrees $p, p-1, .., 0$. While generating closed forms of sequential degrees $k = p, k = p-1, .., k = 0$ the pseudostructures of dimensions $(n+1-k)$: $1, ..., n+1$ are obtained. As a result of transition to the exact closed form of zero degree the metric structure of the dimension $n+1$ is obtained. Under influence of external action (and in the presence of degrees of freedom) the material system can transfer the initial inertial space into the space of the dimension $n+1$.

Sections of the cotangent bundles (Yang-Mills fields), cohomologies by de Rham, singular cohomologies, pseudo-Riemannian and pseudo-Euclidean spaces, and others are examples of psedustructures and spaces that are formed by pseudostructures. Euclidean and Riemannian spaces are examples of metric manifolds that are obtained when going to the exact forms.

What can be said about the pseudo-Riemannian manifold and Riemannian space?

The distinctive property of the Riemannian manifold is an availability of the curvature. This means that the metric form commutator of the third degree is nonzero. Hence, the commutator of the evolutionary form of third degree ($p = 3$), which involves into itself the metric form commutator, is not equal to zero. That is, the evolutionary form that enters into the evolutionary relation is unclosed, and the relation is nonidentical one.

When realizing pseudostructures of the dimensions $1, 2, 3, 4$ and obtaining the closed inexact forms of the degrees $k = 3, k = 2, k = 1, k = 0$ the pseudo-Riemannian space is formed, and the transition to the exact form of zero degree corresponds to the transition to the Riemannian space.

It is well known that while obtaining the Einstein equations it was suggested that there are fulfilled the conditions [6,18]: 1) the Bianchi identity is satisfied, 2) the coefficients of connectedness are symmetric, 3) the condition that the coefficients of connectedness are the Christoffel symbols, and 4) an existence of the transformation under which the coefficients of connectedness vanish. These conditions are the conditions of realization of degenerate transformations for nonidentical relations obtained from the evolutionary nonidentical relation with evolutionary form of the degree $p = 3$ and after going to the identical relations. In this case to the Einstein equation the identical relations with forms of the first degree are assigned.

Conclusion

Results of the analysis carried out have shown the following.

Invariant and covariant properties of closed exterior and dual forms, which correspond to the conservation laws for physical fields, make up the foundations of field theories. The field theories operators are built on the basis of gauge transformations of closed exterior forms. Properties of closed exterior and dual forms explicitly or implicitly manifest themselves essentially in all formalisms of field theories. The degrees of closed exterior forms establish the classification of physical fields and interactions, and this discloses an internal connection between various physical fields and a common basis of corresponding field theories. This shows that the theory of closed exterior forms can be useful in establishing the unified field theory.

Evolutionary forms, which are obtained from the equations of balance conservation laws for material media, answer the question of how are realized the closed exterior forms that correspond to field theories.

This explains the process of originating physical fields and gives the answer to many questions of field theories.

Firstly, this shows that physical fields are generated by material media. The conservation laws for material media, i.e. the balance conservation laws for energy, linear momentum, angular momentum, and mass, which are noncommutative ones, play a controlling role in these processes. This is precisely the noncommutativity of the balance conservation laws produced by external actions onto material system, which is a moving force of evolutionary processes leading to emergency of physical structures (to which exact conservation laws are assigned). And thus the causality of physical processes and phenomena is explained. Since physical fields are made up by discrete physical structures, this points to a quantum character of field theories.

Secondly, it becomes clear a connection of field theory with the equations of mathematical physics describing physical processes in material media. The postulates, which field theories are built on, are the closure conditions for exterior forms obtained from the evolutionary forms connected with these equations.

The connection between closed exterior forms corresponding to field theories and the evolutionary forms obtained from the equations for material media discloses a meaning of the field theory parameters. They relate to the number (p) of interacting noncommutative balance conservation laws and to the degrees (k) of closed exterior forms realized. Hence it arises a possibility to classify physical fields and interactions according to the parameters p and k.

The results obtained on the basis of the theory of skew-symmetric differential forms do not contradict to any physical theories. And yet the methodical results of this theory enable one to understand internal connections between physical fields, between physical fields and material media, between field theories and the equations of mathematical physics, to understand a mechanism of emergency of physical structures and the causality of physical processes and phenomena.

The theory of skew-symmetric differential forms, which unites the theory of closed exterior forms constituting the basis of field theories and the theory of evolutionary forms generating closed inexact exterior forms, can serve as an approach to the general field theory. Such a theory enables one not only to describe physical fields, but also shows how the physical fields are produced, what generates them, and what is a cause of these processes.

Below in Appendices the example of qualitative investigation of the solutions to differential equations and the analysis of the principles of thermodynamics, the equations for gas dynamic system and the equations of electromagnetic field with the help of skew-symmetric differential forms are presented.

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Appendix 1

Qualitative investigation of the solutions of differential equations

The presented method of investigating the solutions to differential equations is not new. Such an approach was developed by Cartan [19] in his analysis of the integrability of differential equations. Here this approach is outlined to demonstrate the role of skew-symmetric differential forms.

The basic idea of the qualitative investigation of the solutions to differential equations can be clarified by the example of the first-order partial differential equation.

Let

$$F(x^i,\, u,\, p_i) = 0, \quad p_i \,=\, \partial u/\partial x^i \tag{A1.1}$$

be the partial differential equation of the first order. Let us consider the functional relation

$$du \,=\, \theta \tag{A1.2}$$

where $\theta \,=\, p_i\,dx^i$ (the summation over repeated indices is implied). Here $\theta \,=\, p_i\,dx^i$ is the differential form of the first degree.

The specific feature of functional relation (A1.2) is that in the general case this relation turns out to be nonidentical.

The left-hand side of this relation involves a differential, and the right-hand side includes the differential form $\theta \,=\, p_i\,dx^i$. For this relation to be identical, the differential form $\theta \,=\, p_i\,dx^i$ must be a differential as well (like the left-hand side of relation (A1.2)), that is, it has to be a closed exterior differential form. To do this it requires the commutator $K_{ij} = \partial p_j/\partial x^i - \partial p_i/\partial x^j$ of the differential form θ has to vanish.

In general case, from equation (A1.1) it does not follow (explicitly) that the derivatives $p_i \,=\, \partial u/\partial x^i$, which obey to the equation (and given boundary or initial conditions of the problem), make up a differential. Without any supplementary conditions the commutator of the differential form θ defined as $K_{ij} = \partial p_j/\partial x^i - \partial p_i/\partial x^j$ is not equal to zero. The form $\theta \,=\, p_i\,dx^i$ occurs to be unclosed and is not a differential like the left-hand side of relation (A1.2). Functional relation (A1.2) appears to be nonidentical: the left-hand side of this relation is a differential, but the right-hand side is not a differential. (The skew-symmetric differential form $\theta \,=\, p_i\,dx^i$, which enters into functional relation (A1.2), is the example of evolutionary skew-symmetric differential forms.)

The nonidentity of functional relation (A1.2) points to a fact that without additional conditions derivatives of the initial equation do not make up a differential. This means that the corresponding solution to the differential equation u will not be a function of x^i. It will depend on the commutator of the form θ, that is, it will be a functional.

To obtain the solution that is a function (i.e., derivatives of this solution form a differential), it is necessary to add the closure condition for the form $\theta \,=\, p_i dx^i$ and for the dual form (in the present case the functional F plays a role of the form dual to θ):

$$\begin{cases} dF(x^i,\, u,\, p_i) \,=\, 0 \\ d(p_i\,dx^i) \,=\, 0 \end{cases} \tag{A1.3}$$

If we expand the differentials, we get a set of homogeneous equations with respect to dx^i and dp_i (in the $2n$-dimensional space – initial and tangential):

$$\begin{cases} \left(\dfrac{\partial F}{\partial x^i} \,+\, \dfrac{\partial F}{\partial u}\,p_i\right)dx^i \,+\, \dfrac{\partial F}{\partial p_i}\,dp_i \,=\, 0 \\ dp_i\,dx^i \,-\, dx^i\,dp_i \,=\, 0 \end{cases} \tag{A1.4}$$

The solvability conditions for this system (vanishing of the determinant composed of coefficients at dx^i, dp_i) have the form:

$$\frac{dx^i}{\partial F/\partial p_i} = \frac{-dp_i}{\partial F/\partial x^i + p_i \partial F/\partial u} \qquad (A1.5)$$

These conditions determine an integrating direction, namely, a pseudostructure, on which the form $\theta = p_i\, dx^i$ occurs to be closed one, i.e. it becomes a differential, and from relation (A1.2) the identical relation is produced. If conditions (A1.5), that may be called the integrability conditions, are satisfied, the derivatives constitute a differential $\delta u = p_i dx^i = du$ (on the pseudostructure), and the solution becomes a function. Just such solutions, namely, functions on pseudostructures formed by the integrating directions, are so-called generalized solutions. The derivatives of the generalized solution constitute the exterior form, which is closed on pseudostructure.

Since the functions that are generalized solutions are defined only on pseudostructures, they have discontinuities in derivatives in the directions being transverse to pseudostructures. The order of derivatives with discontinuities is equal to the exterior form degree. If the form of zero degree is involved in the functional relation, the function itself, being a generalized solution, will have discontinuities.

If we find the characteristics of equation (A1.1), it appears that conditions (A1.5) are equations for characteristics [20]. That is, the characteristics are examples of the pseudostructures on which the derivatives of differential equation constitute closed forms and the solutions turn out to be functions (generalized solutions).

Here it is worth noting that coordinates of the equations for characteristics are not identical to independent coordinates of the initial space on which equation (A1.1) is defined. The transition from initial space to characteristic manifold appears to be a *degenerate* transformation, namely, the determinant of the system of equations (A1.4) becomes zero. The derivatives of equation (A1.1) are transformed from tangent space to cotangent one. The transition from the tangent space, where the commutator of the form θ is nonzero (the form is unclosed, the derivatives do not form a differential), to the characteristic manifold, namely, the cotangent space, where the commutator becomes equal to zero (a closed exterior form is formed, i.e. the derivatives make up a differential), is the example of degenerate transformation.

The solutions to all differential equations have similar functional properties. And, if the order of differential equation is k, the functional relation with k-degree form corresponds to this equation. For ordinary differential equations the commutator is produced at the expense of the conjugacy of derivatives of the functions desired and those of the initial data (the dependence of the solution on the initial data is governed by the commutator).

In a similar manner one can also investigate the solutions to a system of partial differential equations and the solutions to ordinary differential equations (for which the nonconjugacy of desired functions and initial conditions is examined).

It can be shown that the solutions to equations of mathematical physics, on which no additional external conditions are imposed, are functionals. The solutions prove to be exact only under realization of additional requirements, namely, the conditions of degenerate transformations like vanishing determinants, Jacobians and so on, that define integral surfaces. Characteristic manifolds, the envelopes of characteristics, singular points, potentials of simple and double layers, residues and others are the examples of such surfaces.

The dependence of the solution on the commutator can lead to instability of the solution. Equations that do not provided with the integrability conditions (the conditions such as, for example, the characteristics, singular points, integrating factors and others) may have unstable solutions. Unstable solutions appear in the case when additional conditions are not realized and no exact solutions (their derivatives form a differential) are formed. Thus, the solutions to the equations of elliptic type can be unstable.

Investigation of nonidentical functional relations lies at the basis of the qualitative theory of differential equations. It is well known that the qualitative theory of differential equations is based on the analysis of unstable solutions and integrability conditions. From the functional relation it follows that the dependence of the solution on the commutator leads to instability, and the closure conditions of the forms constructed by derivatives are integrability conditions. One can see that the problem of unstable solutions and integrability conditions appears, in fact, to be reduced to the question of under what conditions the identical relation for closed form is produced from the nonidentical relation that corresponds to the relevant differential equation (the relation such as (A1.2)), the identical relation for closed form is produced. In other words, whether or not the solutions are functionals? This is, the analysis of the correctness of setting the problems of mathematical physics is reduced to the same question.

Here the following should be emphasized. When the degenerate transformation from the initial nonidentical functional relation is performed, an integrable identical relation is obtained. As the result of integrating, one obtains a relation that contains exterior forms of less by one degree and which once again proves to be (in the general case without additional conditions) nonidentical. By integrating the functional relations obtained sequentially (it is possible only under realization of the degenerate transformations) from the initial functional relation of degree k one can obtain $(k+1)$ functional relations each involving exterior forms of one of degrees: k, $k-1$, ...0. In particular, for the first-order partial differential equation it is also necessary to analyze the functional relation of zero degree.

Thus, application of skew-symmetric differential forms allows one to reveal the functional properties of the solutions to differential equations.

Analysis of field equations

Field theory is based on the conservation laws. The conservation laws are described by the closure conditions of the exterior differential forms. It is evident that the solutions to the equations of field theory describing physical fields can be only generalized solutions, which correspond to closed exterior differential forms.

The generalized solutions can have a differential equation, which is subject to the additional conditions.

Let us consider what equations are obtained in this case.

Return to equation (A1.1). Assume that the equation does not explicitly depend on u and is resolved with respect to some variable, for example t, that is, it has the form of

$$\frac{\partial u}{\partial t} + E(t,\, x^j,\, p_j) = 0, \quad p_j = \frac{\partial u}{\partial x^j} \qquad (A1.6)$$

Then integrability conditions (A1.5) (the closure conditions of the differential form $\theta = p_i dx^i$ and the corresponding dual form) can be written as (in this case $\partial F/\partial p_1 = 1$)

$$\frac{dx^j}{dt} = \frac{\partial E}{\partial p_j}, \quad \frac{dp_j}{dt} = -\frac{\partial E}{\partial x^j} \qquad (A1.7)$$

These are the characteristic relations for equation (A1.6). As it is well known, the canonical relations have just such a form.

As a result we conclude that the canonical relations are the characteristics of equation (A1.6) and the integrability conditions for this equation.

The canonical relations obtained from the closure condition of the differential form $\theta = p_i dx^i$ and the corresponding dual form, are the examples of the identical relation of the theory of exterior differential forms.

Equation (A1.6) provided with the supplementary conditions, namely, the canonical relations (A1.7), is called the Hamilton-Jacobi equation [20]. In other words, the equation whose derivatives obey the canonical relation is referred to as the Hamilton-Jacobi equation. The derivatives of this equation form the differential, i.e. the closed exterior differential form: $\delta u = (\partial u/\partial t)\, dt + p_j\, dx^j = -E\, dt + p_j\, dx^j = du$.

The equations of field theory belong to this type.

$$\frac{\partial s}{\partial t} + H\left(t,\, q_j,\, \frac{\partial s}{\partial q_j}\right) = 0, \quad \frac{\partial s}{\partial q_j} = p_j \qquad (A1.8)$$

where s is the field function for the action functional $S = \int L\, dt$. Here L is the Lagrange function, H is the Hamilton function: $H(t,\, q_j,\, p_j) = p_j\, \dot{q}_j - L$, $p_j = \partial L/\partial \dot{q}_j$. The closed form $ds = -\,H\, dt + p_j\, dq_j$ (the Poincare invariant) corresponds to equation (A1.8).

A peculiarity of the degenerate transformation can be considered by the example of the field equation.

Here the degenerate transformation is a transition from the Lagrange function to the Hamilton function. The equation for the Lagrange function, that is the Euler variational equation, was obtained from the condition $\delta S = 0$, where S is the action functional. In the real case, when forces are nonpotential or couplings are nonholonomic, the quantity δS is not a closed form, that is, $d\,\delta S \neq 0$. But the Hamilton function is obtained from the condition $d\,\delta S = 0$ which is the closure condition for the form δS. The transition from the Lagrange

function L to the Hamilton function H (the transition from variables q_j, $\dot{q}_j$ to variables q_j, $p_j = \partial L/\partial \dot{q}_j$) is a transition from the tangent space, where the form is unclosed, to the cotangent space with a closed form. This transition is a degenerate one.

The invariant field theories used only nondegenerate transformations that conserve a differential. There exists a relation between nondegenerate transformations and degenerate transformations. In the case under consideration the degenerate transformation is a transition from the tangent space $(q_j,\, \dot{q}_j))$ to the cotangent (characteristic) manifold $(q_j,\, p_j)$, but the nondegenerate transformation is a transition from one characteristic manifold $(q_j,\, p_j)$ to another characteristic manifold $(Q_j,\, P_j)$. {The formula of canonical transformation can be written as $p_j dq_j = P_j dQ_j + dW$, where W is the generating function}.

Appendix 2
The analysis of balance conservation laws for thermodynamic and gas dynamic systems and for the system of charged particles

Thermodynamic systems

The thermodynamics is based on the first and second principles of thermodynamics that were introduced as postulates [21]. The first principle of thermodynamics, which can be written in the form

$$dE\,+\,dw\,=\,\delta Q \qquad (A2.1)$$

follows from the balance conservation laws for energy and linear momentum (but not only from the conservation law for energy). This is analogous to the evolutionary relation for the thermodynamic system. Since δQ is not a differential, relation (A2.1) which corresponds to the first principle of thermodynamics, as well as the evolutionary relation, appears to be a nonidentical relation. This points to a noncommutativity of the balance conservation laws (for energy and linear momentum) and to a nonequilibrium state of the thermodynamic system.

If condition of the integrability be satisfied, from the nonidentical evolutionary relation, which corresponds to the first principle of thermodynamics, it follows an identical relation. It is an identical relation that corresponds to the second principle of thermodynamics.

If $dw\,=\,p\,dV$, there is the integrating factor θ (a quantity which depends only on the characteristics of the system), where $1/\theta\,=\,pV/R$ is called the temperature T [21]. In this case the form $(dE\,+\,p\,dV)/T$ turns out to be a differential (interior) of some quantity that referred to as entropy S:

$$(dE\,+\,p\,dV)/T\,=\,dS \qquad (A2.2)$$

If the integrating factor $\theta = 1/T$ has been realized, that is, relation (A2.2) proves to be satisfied, from relation (A2.1), which corresponds to the first principle of thermodynamics, it follows

$$dS\,=\,\delta Q/T \qquad (A2.3)$$

This is just the second principle of thermodynamics for reversible processes. This takes place when the heat input is the only action onto the system.

If in addition to the heat input the system experiences a certain mechanical action (for example, an influence of boundaries), we obtain

$$dS > \delta Q/T \qquad (A2.4)$$

that corresponds to the second principle of thermodynamics for irreversible processes.

In the case examined above the differential of entropy (rather than entropy itself) becomes a closed form. {In this case entropy manifests itself as the thermodynamic potential, namely, the function of state. To the pseudostructure there corresponds the state equation that determines the temperature dependence on the thermodynamic variables}.

For entropy to be a closed form itself, one more condition must be realized. Such a condition could be the realization of the integrating direction, an example of that is the speed of sound: $a^2 = \partial p/\partial\rho = \gamma p/\rho$. In this case it is valid the equality $ds = d(p/\rho^\lambda) = 0$ from which it follows that entropy $s = p/\rho^\lambda =$ const is a closed form (of zero degree). {However it does not mean that a state of the gaseous system is identically isoentropic. Entropy is constant only along the integrating direction (for example, on the adiabatic curve or on the front of the sound wave), whereas in the direction normal to the integrating direction the normal derivative of entropy has a break}.

Gas dynamical systems

We take the simplest gas dynamical system, namely, a flow of ideal (inviscous, heat nonconductive) gas [13].

Assume that the gas (the element of gas dynamic system) is a thermodynamic system in the state of local equilibrium (whenever the gas dynamic system itself may be in nonequilibrium state), that is, it is satisfied the relation [21]

$$Tds = de + pdV \qquad (A2.5)$$

where T, p and V are the temperature, the pressure and the gas volume, s and e are entropy and internal energy per unit volume.

Let us introduce two frames of reference: an inertial one that is not connected with material system and an accompanying frame of reference that is connected with the manifold formed by the trajectories of the material system elements.

The equation of the balance conservation law of energy for ideal gas can be written as [13]

$$\frac{Dh}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = 0 \qquad (A2.6)$$

where D/Dt is the total derivative with respect to time (if to denote the spatial coordinates by x_i and the velocity components by u_i, $D/Dt = (\partial/\partial t + u_i\partial/\partial x_i)$. Here $\rho = 1/V$ and h are respectively the mass and the entalpy densities of the gas.

Expressing entalpy in terms of internal energy e using the formula $h = e + p/\rho$ and using relation (A2.5), the balance conservation law equation (A2.6) can be put to the form

$$\frac{Ds}{Dt} = 0 \qquad (A2.7)$$

And respectively, the equation of the balance conservation law for linear momentum can be presented as [13,22]

$$\operatorname{grad} s = (\operatorname{grad} h_0 + \mathbf{U} \times \operatorname{rot}\mathbf{U} - \mathbf{F} + \partial\mathbf{U}/\partial t)/T \qquad (A2.8)$$

where $\mathbf{U}$ is the velocity of the gas particle, $h_0 = (\mathbf{U} \cdot \mathbf{U})/2 + h$, $\mathbf{F}$ is the mass force. The operator *grad* in this equation is defined only in the plane normal to the trajectory. [Here it was tolerated a certain incorrectness. Equations (A2.7), (A2.8) are written in different forms. This is connected with difficulties when deriving these equations themselves. However, this incorrectness will not effect on results of the qualitative analysis of the evolutionary relation obtained from these equations.]

Since the total derivative with respect to time is that along the trajectory, in the accompanying frame of reference equations (A2.7) and (A2.8) take the form:

$$\frac{\partial s}{\partial \xi^1} = 0 \qquad (A2.9)$$

$$\frac{\partial s}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, ... \qquad (A2.10)$$

where ξ^1 is the coordinate along the trajectory, $\partial s/\partial \xi^\nu$ is the left-hand side of equation (A2.8), and A_ν is obtained from the right-hand side of relation (A2.8).

{In the common case when gas is nonideal equation (A2.9) can be written in the form

$$\frac{\partial s}{\partial \xi^1} = A_1 \qquad (A2.11)$$

where A_1 is an expression that depends on the energetic actions (transport phenomena: viscous, heat-conductive). In the case of ideal gas $A_1 = 0$ and equation (A2.12) transforms into (A2.9). In the case of the viscous heat-conductive gas described by a set of the Navier-Stokes equations, in the inertial frame of reference the expression A_1 can be written as [13]

$$A_1 = \frac{1}{\rho}\frac{\partial}{\partial x_i}\left(-\frac{q_i}{T}\right) - \frac{q_i}{\rho T}\frac{\partial T}{\partial x_i} + \frac{\tau_{ki}}{\rho}\frac{\partial u_i}{\partial x_k} \qquad (A2.12)$$

Here q_i is the heat flux, τ_{ki} is the viscous stress tensor. In the case of reacting gas extra terms connected with the chemical nonequilibrium are added [13].}

Equations (A2.9) and (A2.10) can be convoluted into the relation

$$ds = A_\mu d\xi^\mu \qquad (A2.13)$$

where $A_\mu d\xi^\mu = \omega$ is the first degree differential form (here $A_1 = 0$,$\mu = 1$, ν).

Relation (A2.13) is the evolutionary relation for gas dynamic system (in the case of local thermodynamic equilibrium). Here $\psi = s$. {It worth notice that in the evolutionary relation for thermodynamic system the dependence of entropy on thermodynamic variables is investigated (see relation (A2.5)), whereas in the evolutionary relation for gas dynamic system the entropy dependence on the space-time variables is considered}.

Relation (A2.13) appears to be nonidentical. To make it sure that this is true one must inspect the commutator of the form ω.

Without accounting for terms that are connected with a deformation of the manifold formed by the trajectories the commutator can be written as

$$K_{1\nu} = \frac{\partial A_\nu}{\partial \xi^1} - \frac{\partial A_1}{\partial \xi^\nu}$$

From the analysis of the expression A_ν and with taking into account that $A_1 = 0$ one can see that terms that are related to the multiple connectedness of the flow domain (the second term of equation (A2.8)), the nonpotentiality of the external forces (the third term in (A2.8)) and the nonstationarity of the flow (the forth term in (A2.8)) contribute into the commutator. {The terms connected with transport phenomena and physical and chemical processes will contribute into the commutator (see expression (A2.12)).}

Since the commutator of the form ω is nonzero, it is evident that the form ω proves to be unclosed. This means that relation (A2.13) cannot be an identical one.

Nonidentity of the evolutionary relation points to the nonequilibrium state and the development of the gas dynamic instability. Since the nonequilibrium state is produced by internal forces that are described by the commutator of the form ω, it becomes evident that the cause of the gas dynamic instability is something that contributes into the commutator of the form ω.

One can see (see (A2.8)) that the development of instability is caused by not a simply connectedness of the flow domain, nonpotential external (for each local domain of the gas dynamic system) forces, a nonstationarity of the flow.

All these factors lead to emergency of internal forces, that is, to nonequilibrium state and to development of various types of instability. {Transport phenomena and physical and chemical processes also lead to emergency of internal forces and to development of instability.}

And yet for every type of instability one can find the appropriate term giving contribution to the evolutionary form commutator, which is responsible for this type of instability. Thus, there is an unambiguous connection between the type of instability and the terms that contribute to the evolutionary form commutator in the evolutionary relation. {In the general case one has to consider the evolutionary relations that correspond to the balance conservation laws for angular momentum and mass as well.}

As it was shown above, under realization of additional degrees of freedom it can take place the transition from the nonequilibrium state to the locally equilibrium one, and this process is accompanied by emergency of physical structures.

The gas dynamic formations that correspond to these physical structures are shocks, shock waves, turbulent pulsations and so on. Additional degrees of freedom are realized as the condition of the degenerate transformation, namely, vanishing of determinants, Jacobians of transformations, etc. These conditions specify the integral surfaces (pseudostructures): the characteristics (the determinant of coefficients at the normal derivatives vanishes), the singular points (Jacobian is equal to zero), the envelopes of characteristics of the Euler equations and so on. Under crossing throughout the integral surfaces the gas dynamic functions or their derivatives undergo the breaks.

Let as analyze which types of instability and what gas dynamic formation can originate under given external action.

1). *Shock, break of diaphragm and others.* The instability originates because of nonstationarity. The last term in equation (A2.8) gives a contribution into the commutator. In the case of ideal gas whose flow is described by equations of the hyperbolic type the transition to the locally equilibrium state is possible on the characteristics and their envelopes. The corresponding structures are weak shocks and shock waves.

2).*Flow of ideal (inviscous, heat nonconductive) gas around bodies Action of nonpotential forces.* The instability develops because of the multiple connectedness of the flow domain and a nonpotentiality of the body forces. The contribution into the commutator comes from the second and third terms of the right-hand side of equation (A2.8). Since the gas is ideal one and $\partial s/\partial \xi^1 = A_1 = 0$, that is, there is no contribution into the each fluid particle, an instability of convective type develops. For $U > a$ (U is the velocity of the gas particle, a is the speed of sound) a set of equations of the balance conservation laws belongs to the hyperbolic type and hence the transition to the locally equilibrium state is possible on the characteristics and on the envelopes of characteristics as well, and weak shocks and shock waves are the structures of the system. If $U < a$ when the equations are of elliptic type, such a transition is possible only at singular points. The structures emerged due to a convection are of the vortex type. Under long acting the large-scale structures can be produced.

3. *Boundary layer.* The instability originates due to the multiple connectness of the domain and the transport phenomena (an effect of viscosity and thermal conductivity). Contributions into the commutator produce the second term in the right-hand side of equation (A2.8) and the second and third terms in expression (A2.12). The transition to the locally equilibrium state is allowed at singular points. because in this case $\partial s/\partial \xi^1 = A_1 \neq 0$, that is, the external exposure acts onto the gas particle separately, the development of instability and the transitions to the locally equilibrium state are allowed only in an individual fluid particle. Hence, the structures emerged behave as pulsations. These are the turbulent pulsations.

{Studying the instability on the basis of the analysis of entropy behavior was carried out in the works by Prigogine and co-authors [23]. In that works entropy was considered as the thermodynamic function of state (though its behavior along the trajectory was analyzed). By means of such state function one can trace the development (in gas fluxes) of the hydrodynamic instability

only. To investigate the gas dynamic instability it is necessary to consider entropy as the gas dynamic state function, i.e. as a function of the space-time coordinates. Whereas for studying the thermodynamic instability one has to analyze the commutator constructed by the mixed derivatives of entropy with respect to the thermodynamic variables, for studying the gas dynamic instability it is necessary to analyze the commutators constructed by the mixed derivatives of entropy with respect to the space-time coordinates.}

Electromagnetic field

The system of charged particles is a material medium, which generates electromagnetic field.

If to use the Lorentz force $\mathbf{F} = \rho(\mathbf{E} + [\mathbf{U}\times\mathbf{H}]/c)$, the local variation of energy and linear momentum of the charged matter (material system) can be written respectively as [10]: $\rho(\mathbf{U}\cdot\mathbf{E})$, $\rho(\mathbf{E} + [\mathbf{U}\times\mathbf{H}]/c)$. Here ρ is the charge density, $\mathbf{U}$ is the velocity of charged matter. These variations of energy and linear momentum are caused by energetic and force actions and are equal to values of these actions. If to denote these actions by Q^e, $\mathbf{Q}^i$, the balance conservation laws can be written as follows:

$$\rho\,(\mathbf{U}\cdot\mathbf{E}) \,=\, Q^e \tag{$A2.14$}$$

$$\rho\,(\mathbf{E} \,+\, [\mathbf{U}\times\mathbf{H}]/c) \,=\, \mathbf{Q}^i \tag{$A2.15$}$$

After eliminating the characteristics of material system (the charged matter) ρ and $\mathbf{U}$ by using the Maxwell-Lorentz equations [10], the left-hand sides of equations (A2.14), (A2.15) can be expressed only in terms of the strengths of electromagnetic field, and then one can write equations (A2.14), (A2.15) as

$$c\,\mathrm{div}\mathbf{S} \,=\, -\frac{\partial}{\partial t}\,I \,+\, Q^e \tag{$A2.16$}$$

$$\frac{1}{c}\,\frac{\partial}{\partial t}\,\mathbf{S} \,=\, \mathbf{G} \,+\, \mathbf{Q^i} \tag{$A2.17$}$$

where $\mathbf{S} = [\mathbf{E}\times\mathbf{H}]$ is the Pointing vector, $I = (E^2 + H^2)/c$, $\mathbf{G} = \mathbf{E}\,\mathrm{div}\mathbf{E} + \mathrm{grad}(\mathbf{E}\cdot\mathbf{E}) - (\mathbf{E}\cdot\mathrm{grad})\mathbf{E} + \mathrm{grad}(\mathbf{H}\cdot\mathbf{H}) - (\mathbf{H}\cdot\mathrm{grad})\mathbf{H}$.

Equation (A2.16) is widely used while describing electromagnetic field and calculating energy and the Pointing vector. But equation (A2.17) does not commonly be taken into account. Actually, the Pointing vector $\mathbf{S}$ must obey two equations that can be convoluted into the *relation*

$$d\mathbf{S} = \omega^{\mathbf{2}} \tag{$A2.18$}$$

Here $d\mathbf{S}$ is the state differential being 2-form and the coefficients of the form ω^2 (the second degree form) are the right-hand sides of equations (A2.16) and (A2.17). It is just the evolutionary relation for the system of charged particles that generate electromagnetic field.

By analyzing the coefficients of the form ω^2 (obtained from equations (A2.16) and (A2.17), one can assure oneself that the form commutator is nonzero. This means that from relation (A2.18) the Pointing vector cannot be found. This points to the fact that there is no such a measurable quantity (a potential).

Under what conditions can the Pointing vector be formed as a measurable quantity?

Let us choose the local coordinates l_k in such a way that one direction l_1 coincides with the direction of the vector $\mathbf{S}$. Because this chosen direction coincides with the direction of the vector $\mathbf{S} = [\mathbf{E} \times \mathbf{H}]$ and hence is normal to the vectors $\mathbf{E}$ and $\mathbf{H}$, one obtains that div$\mathbf{S} = \partial s/\partial l_1$, where S is a module of $\mathbf{S}$. In addition, the projection of the vector $\mathbf{G}$ on the chosen direction turns out to be equal to $-\partial I/\partial l_1$. As a result, after separating from vector equation (A2.17) its projection on the chosen direction equations (A2.16) and (A2.17) can be written as

$$\frac{\partial S}{\partial l_1} = -\frac{1}{c}\frac{\partial I}{\partial t} + \frac{1}{c}Q^e \qquad (A2.19)$$

$$\frac{\partial S}{\partial t} = -c\,\frac{\partial I}{\partial l_1} + c\mathbf{Q}'^i \qquad (A2.20)$$

$$0 = -\mathbf{G}'' - c\mathbf{Q}''^i$$

Here the prime relates to the direction l_1, double primes relate to the other directions. Under the condition $dl_1/dt = c$ from equations (A2.19) and (A2.20) it is possible to obtain the relation in differential forms

$$\frac{\partial S}{\partial l_1}\,dl_1 + \frac{\partial S}{\partial t}\,dt = -\left(\frac{\partial I}{\partial l_1}\,dl_1 + \frac{\partial I}{\partial t}\,dt\right) + (Q^e\,dt + \mathbf{Q}'^i\,dl_1) \qquad (A2.21)$$

Because the expression within the second braces in the right-hand side is not a differential (the energetic and force actions have different nature and cannot be conjugated), one can obtain a closed form only if this term vanishes:

$$(Q^e\,dt + \mathbf{Q}'^i\,dl_1) = 0 \qquad (A2.22)$$

that is possible only discretely (rather than identically).

In this case $dS = 0$, $dI = 0$ and the modulus of the Pointing vector S proves to be a closed form, i.e. a measurable quantity. The integrating direction (the pseudostructure) will be

$$-\frac{\partial S/\partial t}{\partial S/\partial l_1} = \frac{dl_1}{dt} = c \qquad (A2.23)$$

The quantity I is the second dual invariant.

Thus, the constant c entered into the Maxwell equations is defined as the integrating direction.

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